Title: Volume of Pyramid

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Abstract:

The subject of this thesis is Hilbert's third problem.

In the first chapter we follow it's roots back to Euclid's *Elements*. We focus in particular on the theorem that *triangular pyramids of equal altitudes are to each other as their bases*. We also discuss analogous statements for triangles, parallelograms and parallelepipeds. We point out the way in which the issues of content and volume of geometrical figures were approached in Greek mathematics.

In the second chapter we present the historical background of Hilbert's third problem. We outline the development of methods of it's solution – from M. Dehn's first answer in 1901 to the abstract definition of Dehn invariants as a $\mathbb{R} \otimes_{\mathbb{Z}} \mathbb{R}_{\pi}$-valued functional on the polyhedral group that was introduced by B. Jessen in 1968. Later we construct Dehn invariants and present a thorough solution to the Hilbert's third problem. In the end we sketch out mathematical issues connected to this problem that have been studied in the second half of 20th century.

An illustrative high school exercise on derivation of the volume formula for pyramid by Eudoxus's method of exhaustion is included in the appendix.

Keywords: pyramid, volume, Euclid, Dehn's invariants, Hilbert's Third Problem